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A New Version of Locally Closed Sets in Nano Topological Spaces

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Abstract: The purpose of this paper is to introduce a new class of nano generalized closed sets namely, nano*generalized b-closed sets in a nano topological space, Also we have introduce nano*generalized locally b-closed sets and their characterizations are analyzed.

Keywords: $N*gb$-closed set, $N*GBLC$-closed set, $(N*GBLC)*$-closed set, $(N*GBLC)**$-closed set.

I. INTRODUCTION

Andrijevic[2] introduced a new class of generalized open sets in a topological space, the so called b-open sets. Levine[9] derived the concept of generalized closed sets in topological space. Al Omari and Mohd.Salmi Md.Noorani [3] studied the class of generalized b-closed sets. The notation of nano topology was introduced by Lellis Thivagar[11] which was defined in terms of approximations and boundry regions of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano-closure. Nano gb-closed set was initiated by Dhanis Arul Mary and I.Arockiarani[5]. In this paper we use nano gb-closed set as a tool to introduce a new class of sets called nano*generalized b-closed sets and discuss some of its properties. We also propose the idea of nano*generalized locally b-closed sets and study some of its properties

II.PRELIMINARIES

Definition 2.1[12]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \underset{x\in U}{U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by $X \in U$ 2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \underset{x\in U}{U}\{R(x) : R(X) \cap X \neq \varphi\}$

3. The boundary of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2[11]: If (U,R) is an approximation space and $X, Y \subseteq U$,then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\varphi) = U_R(\varphi) = \varphi$ and $L_R(U) = U_R(U) = U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.3[10]: Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$. Let $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and $(U, \tau_R(X))$ is

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called the nano topological space. $\left[\tau_R(X)\right]^c$ is called as the dual nano topology of $\tau_R(X)$. Elements of $\left[\tau_R(X)\right]^c$ are called as nano closed sets.

Definition 2.4[11]: If $\tau_R(X)$ is the nano topolopy on U with respect to X, then the set $B=\{U, L_R(X), U_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5[11]: If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by $N\operatorname{int}(A)$. That is $N\operatorname{int}(A)$, is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$. That is $Ncl(A)$, is the smallest nano closed set containing A.

Definition2.6 [6]: A subset A of a nano topological space $(U, \tau_R(X))$ is called nano generalized b-closed (briefly, nano gb-closed), if Nbcl(A)⊆V whenever A⊆V and V is nano open in U .

Definition 2.7[11]: Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be

Nano semi opens If $A \subseteq Ncl(N\operatorname{int}(A))$

Nano pre-open if $A \subseteq N\operatorname{int}(Ncl(A))$

Nano α -open if $A \subseteq N\operatorname{int}(Ncl(N\operatorname{int}(A)))$

Nano b-open if $A \subseteq Ncl(N\operatorname{int}(A)) \cup N\operatorname{int}(Ncl(A))$

Nano regular-open if $A = N\operatorname{int}(Ncl(A))$

NSO(U, X), NPO(U, X), NαO(U, X), NBO (U, X) and NRO(U, X) respectively denote the families of all nano semi open, nano pre open, nano α open, nano b-open, nano r-open subsets of U. Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. A is said to be nano semi closed, nano pre -closed and nano α -closed, nano b-closed ,nano regular-closed if its complement is respectively nano semi-open, nano pre-open, nano α -open, nano b-open, nano regular-open.

III. NANO*GENERALIZED b-CLOSED SETS

Definition 3.1: A subset A of a nano topological space $(U, \tau_R(X))$ is called nano*generalized b-closed if $Nbcl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano gb-open in U

Definition 3.2: A subset A of a nano topological space $(U, \tau_R(X))$ is called

(1) nano r-closed if $Nrcl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano gb-open in U.

(2) nano c-closed if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano gb-open in U.

(3) nano p-closed if $Npcl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano gb-open in U.

(4) nano s-closed if $Nscl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano gb-open in U.

(5) nano α -closed if $N\alpha cl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano gb-open in U.

Theorem 3.3:

(a) Every nano r-closed set is nano*generalized b-closed set.

(b) Every nano c-closed set is nano*generalized b-closed set

(c) Every nano closed set is nano*generalized b-closed set

(d) Every nano s- closed set is nano*generalized b-closed set

(e) Every nano p- closed set is nano*generalized b-closed set

(f) Every nano α - closed set is nano *generalized b-closed set.

Proof: (a) Let A be nano r-closed set. Then $Nrcl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano gb-open in U. But $Nbcl(A) \subseteq Nrcl(A)$ whenever $A \subseteq V$, V is nano gb-open in U. Now we have $Nbcl(A) \subseteq V$, V is nano gb-open. Therefore A is nano*generalized b-closed set. Proof is obvious for others

Remark 3.4: The converse of the above theorem need not be true which can be seen from the following examples.

Example 3.5: Let $U=\{a,b,c\}$ with $U/R=\{\{a\},\{b,c\}\}$, $X=\{a,b\}$ Then $\tau_R(X)=\{U, \varphi, \{a\}, \{b,c\}\}$. nano r-closed set : $\{U, \varphi, \{a\}, \{b,c\}\}$ nano*generalized b-closed set:

$\{U,\varphi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$. Here the set $\{a,c\}$ is nano*generalized b-closed set but not r-closed set.

Example 3.6: Let $U=\{a,b,c\}$ with $U/R=\{\{a\},\{b,c\}\}$ and $X=\{a,c\}$. Then the nano topology is defined as $\tau_R(X)=\{U,\varphi,\{a\},\{b,c\}\}$ nano c-closed set: $\{U,\varphi,\{a\},\{b,c\}\}$ nano*generalized b-closed set.: $\{U,\varphi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$. Here the set $\{c\}$ is nano*generalized b-closed set but not c-closed set.

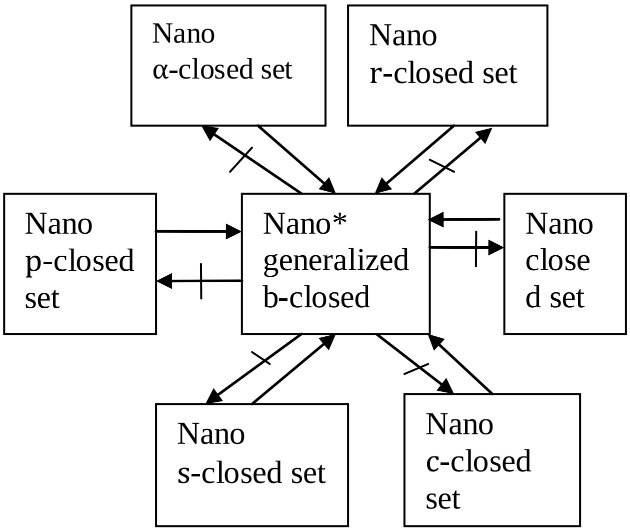
Example 3.7: Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and $X=\{a,b\}$. Then the nano topology is defined as $\tau_R(X)=\{U,\varphi,\{a\},\{a,b,d\},\{b,d\}\}$, nano closed set: $\{U,\varphi,\{c\},\{a,c\},\{b,c,d\}\}$, nano*generalized b-closed sets: $\{U,\varphi,\{a\},\{b\},\{c\},\{d\},\{a,c\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$. Here the set $\{a,c,d\}$ is nano*generalized b-closed set but not nano closed set.

Example 3.8: Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{b\},\{c,d\}\}$ and $X=\{a,c\}$. Then the nano topology is defined as $\tau_R(X)=\{U,\varphi,\{a\},\{c,d\},\{a,c,d\}\}$, nano s-closed set: $\{U,\varphi,\{b\},\{a,b\},\{b,c,d\}\}$ nano* b-closed set: $\{U,\varphi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{b,c,d\},\{a,b,d\}\}$. Here the set $\{b,c\}$ is nano*generalized b-closed set but not nano s-closed set.

Example 3.9: Let $U=\{a,b,c,d\}$ with $U/R=\{\{b\},\{c\},\{a,d\}\}$ and $X=\{b,d\}$. Then the nano topology is defined as $\tau_R(X)=\{U,\varphi,\{b\},\{a,b,d\},\{a,d\}\}$, nano p-closed set: $\{U,\varphi,\{a\},\{c\},\{d\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$, nano*generalized b-closed set: $\{U,\varphi,\{a\},\{b\},\{c\},\{d\},\{a,c\},\{a,d\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$. Here the set $\{b\}$ is nano*generalized b-closed set but not nano p-closed set..

Example 3.10: Let $U=\{a,b,c\}$ with $U/R=\{\{b\},\{a,c\}\}$ and $X=\{\{b,c\}\}$ then the nano topology is defined as $\tau_R(X)=\{U,\varphi,\{b\},\{a,c\}\}$, nano α-closed set: $\{U,\varphi,\{b\},\{a,c\}\}$ nano*generalized b-closed set.: $\{U,\varphi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$. Here the set $\{a,b\}$ is nano*generalized b-closed set but not nano α-closed set.

Remark 3.11:
From the above theorem and examples, we have the following diagrammatic representation:



Theorem 3.12: A set A is nano*generalized b-closed set iff $Nbcl(A)-A$ contains no non-empty nano gb-closed set.

Proof:

Necessity: Let F be a nano gb-closed set in $(U,\tau_R(X))$ such that $F\subseteq Nbcl(A)-A$. Then $A\subseteq X-F$. Since A is nano*generalized b-closed set and $X-F$ is nano gb-open then $Nbcl(A)\subseteq X-F$. That is $F\subseteq X-Nbcl(A)$. So $F\subseteq(X-Nbcl(A))\cap(Nbcl(A)-A)$ Therefore $F=\varphi$.

Sufficiency: Let us assume that $Nbcl(A)-A$ contains no non-empty nano gb-closed set. Let $A\subseteq V$, V is nano gb-open. Suppose that $Nbcl(A)$ is not contained in V, $Nbcl(A)\cap V^C$ is non empty nano gb-closed set of

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$Nbcl(A)-A$ which is contradiction therefore $Nbcl(A)\subseteq V$. Hence A is nano*generalized b-closed set.

Theorem 3.13: If A is nano*generalized b-closed set and $A\subseteq B\subseteq Nbcl(A)$ then B is nano*generalized gb-closed set.

Proof: Let $B\subseteq V$ where V is nano gb-open in $\tau_R(X)$. Then $A\subseteq B$ implies $A\subseteq V$. Since A is nano*generalized b-closed set $Nbcl(A)\subseteq V$. Also $B\subseteq Nbcl(A)$ implies $Nbcl(B)\subseteq Nbcl(A)$. This shows that $Nbcl(B)\subseteq V$ and so B is nano*generalized gb-closed set.

NANO*GENERALIZED b-OPEN SETS

Definition 3.14: A subset A of a nano topological space $(U,\tau_R(X))$ is called nano* gb-open set, if A^c is nano*gb-closed.

Theorem 3.15: A subset $A\subseteq U$ is nano*generalized b-open, if and only if $F\subseteq Nb\text{int}(A)$ whenever F is nano*gb-closed set $F\subseteq A$.

Proof: Let A be nano*generalized b-open set and suppose $F\subseteq A$ where F is nano gb-closed set. Then $U-A$ is a nano*generalized b-closed set contained in the nano gb-open set $U-F$. Hence $Nbcl(U-A)\subseteq U-F$ and $U-Nb\text{int}(A)\subseteq U-F$. Thus $F\subseteq Nb\text{int}(A)$. Conversely, if F is nano*gb-closed set with $F\subseteq Nb\text{int}(A)$ and $F\subseteq A$, then $U-Nb\text{int}(A)\subseteq U-F$. Thus $Nbcl(U-A)\subseteq U-F$. Hence $U-A$ is a nano*generalized b-closed set and A is nano*generalized b-open set.

Theorem 3.16: If A is nano*generalized b-open , V is nano open and $Nb\text{int}(A)\cup A^c\subseteq V$ then V=U.

Proof: Let A be nano*generalized b-open and V is nano gb-open such that $Nb\text{int}(A)\cup A^c\subseteq V$ Then $V^c\subseteq A\cap Nbcl(A^c)\subseteq Nbcl(A^C)-A^C$. Since A^c is nano*generalized b-closed, $Nbcl(A^c)-A^c$ cannot contain any non-empty nano gb-closed set. But V^C is a nano closed subset of $Nbcl(A^c)-A^c$. Therefore, $V^c=\varphi$. That is V=U.

Theorem 3.17: If $Nb\text{int}\,A\subseteq B\subseteq A$ and A is nano*gb-open then B is nano*gb-open.

Proof: Given $Nb\text{int}\,A\subseteq B\subseteq A$ implies $X-A\subset X-B\subset X-Nb\text{int}(A)$. Then $X-A\subset X-B\subset Nbcl(X-A)$ .Since $X-A$ is nano*gb-closed by theorem 3.12 $X-B$ is nano*gb-closed and hence B is nano*gb-open.

Theorem3.18: If $A\subset X$ is nano*gb-closed then $Nbcl(A)-A$ is nano*gb-open.

Proof: Let A be nano*gb-closed. Let F be nano*gb-closed set such that $F\subseteq Nbcl(A)-A$. Then by theorem 3.20 , $F=\phi$, so $F\subseteq Nb\text{int}(Nbcl(A)-A)$. This shows that $Nbcl(A)-A$ is nano*gb-open.

IV.NANO * GENERALIZED b- LOCALLY CLOSED SETS

Definition 4.1: A subset A of $(U,\tau_R(X))$ is called nano*generalized b- locally closed set (briefly $N*gblc$), if $A=G\cap F$ where G is $N*gb$ open in $(U,\tau_R(X))$ and F is $N*gb$ closed in $(U,\tau_R(X))$. The collection of all nano* generalized b-locally closed sets of $(U,\tau_R(X))$ will be denoted by $N*GBLC(U,\tau_R(X))$

Definition 4.2: For a subset A of $(U,\tau_R(X))$ $A\in(N*gblc)*(U,\tau_R(X))$ if there exist a $N*gb$ open set G and a nano closed set F of $(U,\tau_R(X))$ respectively, such that $A=G\cap F$.

Definition 4.3: For a subset A of $(U,\tau_R(X))$ $A\in(N*gblc)**(U,\tau_R(X))$ if there exist a nano open set G and $N*gb$ closed set F of $(U,\tau_R(X))$ respectively such that $A=G\cap F$

Theorem 4.4:

(1) Every nano locally closed set is $(N*GBLC)$
(2) Every nano locally closed set is $(N*GBLC)*$
(3) Every nano locally closed set is $(N*GBLC)**$
(4) Every nano $(N*GBLC)*$ is $(N*GBLC)$
(5) Every nano $(N*GBLC)**$ is $(N*GBLC)$

However the converses of the above are not true may be seen by the following examples.

Example 4.5: Let $U=\{a,b,c\}$ $U/R=\{\{a\},\{b,c\}\}$, $X=\{a,c\}$,$\tau_R(X)=\{U,\phi,\{a\},\{b,c\}\}$. Then the NLC-sets are $\{U,\phi,\{a\},\{b,c\}\}$ and the $N*GBLC$ sets are

$\{U,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$. Here $\{a,c\}$ is $N*GBLC$-closed but not nano locally closed.

Example 4.6: Let $U=\{a,b,c\}$ $U/R=\{\{b\},\{a,c\}\}$, $X=\{b,c\}$, $\tau_R(X)=\{U,\phi,\{b\},\{a,c\}\}$. Then the NLC-sets are $\{U,\phi,\{b\},\{a,c\}\}$ and the $(N*GBLC)*$ sets are $\{U,\phi,\{a\},\{b\},\{c\},\{a,c\}\}$ Here $\{a\}$ is $(N*GBLC)*$ closed but not nano locally closed.

Example 4.7: Let $U=\{a,b,c,d\}$ $U/R=\{\{a\},\{b\},\{c,d\}\}$, $X=\{a,c\}$, $\tau_R(X)=\{U,\phi,\{a\},\{c,d\},\{a,c,d\}\}$.Then the NLC sets are $\{U,\phi,\{a\},\{c,d\}\}$ and the $(N*GBLC)**$ sets are $\{U,\phi,\{a\},\{c\},\{d\},\{a,c\},\{c,d\},\{a,d\}\}$. Here $\{c\}$ is $(N*GBLC)**$ closed but not nano locally closed.

Example 4.8: Let $U=\{a,b,c,d\}$ $U/R=\{\{a\},\{c\},\{b,d\}\}$ $X=\{a,b\}$ $\tau_R(X)=\{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$.Then the $(N*GBLC)*$ sets are $\{U,\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,d\},\{a,b,c\}\}$ and the $N*GBLC$ sets are $\{U,\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$ Here $\{a,b,c\}$ is $N*GBLC$-closed but not $(N*GBLC)*$- closed.

Example 4.9: Let $U=\{a,b,c,d\}$ $U/R=\{\{b\},\{c\},\{a,d\}\}$ $X=\{a,b\}$ $\tau_R(X)=\{U,\phi,\{c\},\{b,c\},\{a,c,d\}\}$.Then the $(N*GBLC)**$ sets are $\{U,\phi,\{a\},\{b\},\{d\},\{a,b\},\{a,d\},\{b,d\}\}$ and the $N*GBLC$ sets are $\{U,\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$ Here $\{c\}$ is $N*GBLC$-closed but not $(N*GBLC)**$- closed.

Theorem 4.10: Let A be nano*gb-closed set. Then A is nano b-closed iff $Nbcl(A)-A$ is nano gb-closed.

Proof: Let A be nano*gb-closed set. If A is nano b-closed, then we have $Nbcl(A)-A=\phi$ which is a nano closed set. Conversely, let $Nbcl(A)-A$ be nano gb- closed. Then by Theorem 3.12 $Nbcl(A)-A$ does not contain any non-empty nano closed set. Thus, $Nbcl(A)-A=\phi$. That is Nbcl(A)=A. Therefore A is nano gb-closed

Theorem 4.11:
For a subset A of $(U,\tau_R(X))$ the following are equivalent
(i) $A\in(N*GBLC)*(U,\tau_R(X))$
(ii) $A=p\cap Ncl(A)$ for some N * gb-open set P
(iii) Ncl(A) – A is N* gb closed
(iv) $A\cup(U-Ncl(A))\, is N*gb$ - open

Proof:
$(i)\Rightarrow(ii)$ let $A\in(N*GBLC)*(U,\tau_R(X))$. Then $A=P\cap F$ where P is $N*gb$- open and F is nano closed. Since $A\subseteq P$ and $A\subseteq Ncl(A)$, $A\subseteq P\cap Ncl(A)$. Conversely, Since $A\subseteq F$, $Ncl(A)\subseteq F$, we have $A=P\cap F$ contains $P\cap NCl(A)$ .That is $P\cap NCl(A)\subseteq A$. Therefore we have $A=P\cap NCl(A)$.
$(ii)\Rightarrow(i)$ Since P is $N*gb$-open and $NCl(A)$ is nano-closed, $P\cap NCl(A)\in(N*GBLC)*(U,\tau_R(X))$ by definition (5.2) of $(N*GBLC)*(U,\tau_R(X))$.
$(ii)\Rightarrow(iii)$ $A=P\cap NCl(A)$ implies that $Ncl(A)-A=Ncl(A)\cap P^C$ which is $N*gb$ closed, Since p^c is $N*gb$ closed.
$(iii)\Rightarrow(ii)$ Let $P=[Ncl(A)-A]^C$. Then by assumption, P is $N*gb$ open in $(U,\tau_R(X))$ and $A=P\cap NCl(A)$.
$(iii)\Rightarrow(iv)$
$A\cup(U-Ncl(A))=A\cup(Ncl(A))^c=[Ncl(A)-A]^C$ and by assumption $[Ncl(A)-A]^C$ is $N*gb$-open and $A\cup(U-Ncl(A))$ is $N*gb$-open.
$(iv)\Rightarrow(iii)$ let $P=A\cup(Ncl(A))^C$. Then P^C is $N*gb$- closed and $P^C=Ncl(A)-A$ and therefore $Ncl(A)-A$ is $N*gb$ closed.

Theorem 4.12:
For a subset A of $(U,\tau_R(X))$,the following statements are equivalent.

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(i) $A \in N^*GBLC(U, \tau_R(X))$
(ii) $A = P \cap N^*gb - cl(A)$ for some N^*gb-open set P.
(iii) $N^*gblc(A) - A$ is N^*gb closed
(iv) $A \cup (N^*gb - cl(A))^C$ is N^*gb open.
(v) $A \subseteq N^*gb - \text{int}(A \cup (N^*gb - cl(A))^C)$

Proof:
$(i) \Rightarrow (ii)$ Let $A \in N^*GBLC(U, \tau_R(X))$. Then $A = P \cap F$ where P is N^*gb - open and F is N^*gb -closed. Since $A \subseteq F, N^*gb - cl(A) \subseteq F$ and therefore $P \cap N^*gb - Ncl(A) - A$. Also $A \subseteq P$ and $A \subseteq N^*gb - cl(A)$ implies $A \subseteq P \cap N^*gb - cl(A)$ and therefore $A = P \cap N^*gb - cl(A)$.
$(iv) \Rightarrow (v)$ By assumption, $A \cup (N^*gb - cl(A))^C = N^*gb - \text{int}(A \cup (N^*gb - cl(A))^C)$ and hence $A \subseteq N^*gb - \text{int}(A \cup (N^*gb - cl(A))^C)$.
$(v) \Rightarrow (i)$ By assumption and since $A \subseteq N^*gb - cl(A)$, $A = N^*gb - \text{int}(A \cup (N^*gb - cl(A))^C) \cap N^*gb - cl(A) \in N^*GBLC(U, \tau_R(X))$.

Theorem 4.13: Let A be a subset of $(U, \tau_R(X))$. Then $A \in (N^*GBLC)^{**}(U, \tau_R(X))$ if and only if $A = P \cap N^*gb - cl(A)$ for some nano open set P.

Proof: Let $A \in (N^*GBLC)^{**}(U, \tau_R(X))$. Then $A = P \cap F$ where P is nano open and F is N^*gb-closed. Since $A \subseteq F$, $N^*gb - cl(A) \subseteq F$ Now $A = A \cap N^*gb - cl(A) = P \cap F \cap N^*gb - cl(A) = P \cap N^*gb - cl(A)$.
Here the converse part is trivial.

Corollary 4.14: Let A be a subset of $(U, \tau_R(X))$. If $A \in (N^*GBLC)^{**}(U, \tau_R(X))$, then $N^*gb - cl(A) - A$ is N^*gb-closed and $A \cup (N^*gb - cl(A))^C$ is N^*gb-open.

Proof: Let $A \in (N^*GBLC)^{**}(U, \tau_R(X))$. Then by above theorem, $A = P \cap N^*gb - cl(A)$ for some nano open set P and $N^*gb - cl(A) - A = N^*gb - cl(A) \cap P^C$ is N^*gb - closed in $(U, \tau_R(X))$. If $F = N^*gb - Cl(A) - A$, then $F^C = A \cup (N^*gb - cl(A))^C$ and F^C is N^*gb- open and therefore $A \cup (N^*gb - cl(A))^C$ is N^*gb-open.

REFERENCES

[1] D. Andrijevic "Semi pre-open sets", Mat.Vesnik,38(1986), 24-32.
[2] D. Andrijevic, "On b-open sets", Mat.Vesnik, 48(1996), 59-64.
[3] Ahmad Al-Omari and Mohd. Salmi Md. Noorani, "On Generalized b-closed sets", Bull. Malays. Math. Sci. Soc.(2)32(1)(2009), 19-30.
[4] P. Bhattacharryya and B.K. Lahiri, "Semi generalized closed sets in topology", Indian J. Math.29(1987),no.3,375-382(1988).
[5] Dhanis Arul Mary, I. Arockiarani, "On Semi Pre-closed sets in nano topological spaces", Mathematical Sciences Int. Research Journal, Volume 3,Issue 2(2014), ISSN 2278-8697.
[6] Dhanis Arul Mary, I. Arockiarani, "On nano gb-closed sets in nano topological spaces" IJMA-6(2),Feb-2015.
[7] K. Balachandran, P. Sundaram and H. Maki, "Generalized Locally closed sets and GLC-Continuous Functions", Indian J. Pure Appl.Math., 27(3)(1996),235-244.